The Lebesgue Measure: An Unmeasurable Set

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Abstract. One of the most important creations of Analysis of the last century was Lebesgue Integral, which remarkably extended Riemann Integral, solved within a few years the fundamental problems of Integration Theory which gave a relevant impetus to Functional Analysis, Theory of Differential Equations and Probability Theory. The basic point of this new theory was the introduction of the notion of measurement. The Lebesgue measure of $\mathbb{R}$ is roughly a function whose domain is a subset of $\mathbb{R}$ and whose contradiction is the set of nonnegative real numbers (joined with the symbol $+\infty$). The length of a range, for example, is a measure, say L, defined over all ranges of the real line, such that $L(I) = b - a$, where a and b, $a < b$, are the extremes of the range I and $L(I) = +\infty$ if I is not limited. Now, the L measure is defined for intervals only. It would be interesting to extend this concept to other subsets of the line. In this work, the outer measure of a subset of $\mathbb{R}$ will be defined and, with this measure, the notion of measurable set will be defined. Then the Lebesgue measure is presented. Our main goal is to build a set of real numbers that cannot be measured with the Lebesgue measure. We conclude that even extending the concept of interval length to other subsets does not encompass them all.
Keywords: Lebesgue, Measurable set, Real numbers, Measure.

References

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